

Supplement to Table of the Roots of the Associated Legendre Function with Respect to the Degree

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$\alpha = 5^\circ$

n \ m	0	1	2	3	4	5
1	27.0558	43.4110	58.3605	72.6312	86.4862	100.0549
2	62.7548	79.8943	95.9610	111.3661	126.3129	140.9192
3	98.6640	116.0804	132.6583	148.6530	164.2131	179.4337
4	134.6207	152.1791	169.0528	185.4151	201.3771	217.0153
5	170.5955	188.2402	205.3074	221.9221	238.1701	254.1134
6	206.5789	224.2817	241.4855	258.2847	274.7472	290.9244
7	242.5673	260.3118	277.6172	294.5572	311.1870	327.5495
8	278.5586	296.3346	313.7186	330.7697	347.5337	364.0467
9	314.5519	332.3525	349.7991	366.9401	383.8141	400.4522
10	350.5466	368.3669	385.8645	403.0799	420.0458	436.7893

Table 1.1: Roots λ_{mn} of associated Legendre Functions with respect to the degree for $\alpha = 5^\circ$

$\alpha = 10^\circ$

n \ m	0	1	2	3	4	5
1	13.2756	21.4598	28.9462	36.0956	43.0385	49.8392
2	31.1264	39.6995	47.7402	55.4527	62.9376	70.2535
3	49.0814	57.7918	66.0862	74.0912	81.3905	89.5013
4	67.0599	75.8408	84.2819	92.4693	100.4581	108.2862
5	85.0474	93.8711	102.4083	110.7209	118.8516	126.8311
6	103.0392	111.8917	120.4967	128.9008	137.1380	145.2335
7	121.0334	129.9066	138.5620	147.0360	155.3562	163.5436
8	139.0291	147.9180	156.6123	165.1415	173.5282	181.7904
9	157.0258	165.9268	174.6522	183.2260	191.6673	199.9916
10	175.0231	183.9340	192.6847	201.2954	209.7823	218.1588

Table 1.2: Roots λ_{mn} of associated Legendre Functions with respect to the degree for $\alpha = 10^\circ$

$\alpha = 15^\circ$

n \ m	0	1	2	3	4	5
1	8.6812	14.1446	19.1486	23.9306	28.5764	33.1284
2	20.5832	26.3023	31.6710	36.8237	41.8265	46.7180
3	32.5535	38.3630	43.8987	49.2439	54.4471	59.5394
4	44.5394	50.3952	56.0275	61.4927	66.8272	72.0559
5	56.5312	62.4152	68.1106	73.6583	79.0862	84.4146
6	68.5258	74.4287	80.1688	85.7767	91.2746	96.6794
7	80.5220	86.4386	92.2118	97.8656	103.4182	108.8835
8	92.5191	98.4460	104.2449	109.9350	115.5314	121.0459
9	104.5170	110.4518	116.2711	121.9907	127.6230	133.1783
10	116.5152	122.4565	128.2925	134.0363	139.6987	145.2884

Table 1.3: Roots λ_{mn} of associated Legendre Functions with respect to the degree for $\alpha = 15^\circ$

$\alpha = 35^\circ$

$n \setminus m$	0	1	2	3	4	5
1	3.4260	5.7830	7.9835	10.0881	12.1400	14.1554
2	8.5318	10.9958	13.3258	15.5727	17.7621	19.9086
3	13.6633	16.1619	18.5558	20.8764	23.1427	25.3664
4	18.8008	21.3171	23.7178	26.1165	28.4312	30.7072
5	23.9405	26.4675	28.9274	31.3207	33.6732	35.9875
6	29.0813	31.6156	34.0875	36.5089	38.8882	41.2317
7	34.2227	36.7622	39.2469	41.6858	44.0860	46.4528
8	39.3645	41.9079	44.4024	46.8552	49.2722	51.6578
9	44.5065	47.0530	49.5553	52.0194	54.4501	56.8513
10	49.6487	52.1977	54.7063	57.1797	59.6219	62.0363

Table 1.7: Roots λ_{nm} of associated Legendre Functions with respect to the degree for $\alpha = 35^\circ$

$\alpha = 40^\circ$

$n \setminus m$	0	1	2	3	4	5
1	2.9323	5.0120	6.9442	8.8045	10.6206	12.4060
2	7.4015	9.5619	11.6104	13.5897	15.5207	17.4159
3	11.8921	14.0813	16.1830	18.2239	20.2192	22.1789
4	16.3876	18.5916	20.7241	22.8033	24.8407	26.8442
5	20.8850	23.0979	25.2507	27.3562	29.4234	31.4588
6	25.3832	27.6022	29.7692	31.8940	33.9837	36.0435
7	29.8820	32.1054	34.2830	36.4224	38.5295	40.6087
8	34.3811	36.6078	38.7935	40.9446	43.0657	45.1606
9	38.8804	41.1096	43.3019	45.4624	47.5945	49.7029
10	43.3799	45.6111	47.8087	49.9769	52.1191	54.2381

Table 1.8: Roots λ_{nm} of associated Legendre Functions with respect to the degree for $\alpha = 40^\circ$

$\alpha = 45^\circ$

$n \setminus m$	0	1	2	3	4	5
1	2.5479	4.4053	6.1388	7.8117	9.4472	11.0569
2	6.5222	8.4471	10.2781	12.0508	13.7831	15.4852
3	10.5143	12.4633	14.3389	16.1634	17.9947	19.7060
4	14.5106	16.4719	18.3734	20.2301	22.0516	23.8447
5	18.5083	20.4773	22.3957	24.2745	26.1211	27.9410
6	22.5069	24.4809	26.4113	28.3062	30.1716	32.0120
7	26.5059	28.4835	30.4228	32.3300	34.2101	36.0668
8	30.5051	32.4855	34.4516	36.3486	38.2404	40.1103
9	34.5045	36.4871	38.4791	40.3653	42.2650	44.1458
10	38.5041	40.4884	42.4444	44.3758	46.2853	48.1753

Table 1.9: Roots λ_{nm} of associated Legendre Functions with respect to the degree for $\alpha = 45^\circ$

$\alpha = 20^\circ$

$n \setminus m$	0	1	2	3	4	5
1	6.3832	10.4885	14.2553	17.8983	21.3609	24.7942
2	15.3112	19.6044	27.6398	31.2816	34.9653	38.3112
3	24.2894	28.6492	32.8073	36.8252	40.7386	43.5702
4	33.2750	37.6239	41.3021	46.0084	50.0185	53.3515
5	42.2725	46.6976	50.3633	53.3915	58.2091	63.1697
6	51.2660	55.7668	60.0098	63.3715	67.4632	71.4087
7	60.2644	64.8360	69.0092	73.2831	77.4632	81.5598
8	69.2625	73.7103	78.0622	82.3341	86.5360	90.6795
9	78.2625	82.7145	87.0815	91.3750	95.6044	99.7770
10	87.2612	91.7180	96.0972	100.4086	104.6601	108.8580

Table 1.4: Roots λ_{nm} of associated Legendre Functions with respect to the degree for $\alpha = 20^\circ$

$\alpha = 25^\circ$

$n \setminus m$	0	1	2	3	4	5
1	5.0038	8.2960	11.3238	14.2234	17.0444	19.8112
2	12.1478	15.5864	18.8237	21.9367	24.9634	27.9260
3	19.3308	22.8213	26.1545	29.3782	32.5202	35.5984
4	26.5226	30.0398	33.4285	36.7212	39.9387	43.0954
5	33.7179	37.2513	40.6763	44.0163	47.2875	50.5015
6	40.9148	44.4591	47.9097	51.2883	54.5959	57.8538
7	48.1126	51.6647	55.1344	58.5555	61.8784	65.1711
8	55.3110	58.8690	62.3534	65.7754	69.1434	72.4644
9	62.5097	66.0723	69.5685	73.0073	76.3961	79.7468
10	69.7087	73.2750	76.7808	80.2335	83.6355	87.0038

Table 1.5: Roots λ_{nm} of associated Legendre Functions with respect to the degree for $\alpha = 25^\circ$

$\alpha = 30^\circ$

$n \setminus m$	0	1	2	3	4	5
1	4.0837	6.8354	9.3733	11.8074	14.1778	16.5041
2	10.0386	12.9083	15.6154	18.2219	20.7588	23.2437
3	16.0248	18.9364	21.7210	24.4170	27.0470	29.6254
4	22.0183	24.9514	27.7807	30.5325	33.2235	35.8654
5	28.0120	30.9606	33.8193	36.6093	39.3438	42.0320
6	34.0102	36.9669	39.8463	42.6643	45.4313	48.1549
7	40.0102	42.9715	45.8662	48.7056	51.4979	54.2497
8	46.0089	48.9749	51.8816	54.7378	57.5505	60.3251
9	52.0079	54.9777	57.8938	60.7636	63.5930	66.3866
10	58.0071	60.9798	63.9037	66.7848	69.6281	72.4377

Table 1.6: Roots λ_{nm} of associated Legendre Functions with respect to the degree for $\alpha = 30^\circ$

$\alpha = 65^\circ$

$n \setminus m$	0	1	2	3	4	5
1	1.5988	2.9178	4.1786	5.4106	6.6249	7.8267
2	4.3565	7.0121	8.2907	8.2907	9.5504	10.7958
3	7.1221	8.4829	9.8096	11.1123	12.3972	13.6681
4	9.8895	11.2562	12.5949	13.9126	15.2140	16.5020
5	12.6577	14.0279	15.3745	16.7025	18.0156	19.3162
6	15.4263	16.7988	18.1509	19.4866	20.8084	22.1186
7	18.1951	19.5693	20.9255	22.2669	23.5956	24.9133
8	20.9639	22.3394	23.6989	25.0448	26.3790	27.7029
9	23.7329	25.1094	26.4714	27.8210	29.1596	30.4887
10	26.5019	27.8792	29.2433	30.5959	31.9383	33.2717

Table 1.13: Roots λ_{nm} of associated Legendre Functions with respect to the degree for $\alpha = 65^\circ$

$\alpha = 70^\circ$

$n \setminus m$	0	1	2	3	4	5
1	1.4456	2.6802	3.8605	5.0362	6.1880	7.3318
2	4.4056	5.2646	6.4909	7.6986	8.8890	10.0590
3	6.2766	7.8138	9.0843	10.3022	11.5140	12.7109
4	8.1477	10.4184	11.6683	12.9018	14.1225	15.3329
5	11.7173	12.9917	14.2749	15.4894	16.7192	17.9393
6	14.2882	15.5645	16.8249	18.0723	19.3091	20.5366
7	16.8593	18.1369	19.4005	20.6524	21.8945	23.1280
8	19.4304	20.7090	21.9751	23.2306	24.4770	25.7153
9	22.0017	23.2810	24.5491	25.8075	27.0573	28.2996
10	24.5729	25.8529	27.1226	28.3834	29.6361	30.8818

Table 1.14: Roots λ_{nm} of associated Legendre Functions with respect to the degree for $\alpha = 70^\circ$

$\alpha = 75^\circ$

$n \setminus m$	0	1	2	3	4	5
1	1.3124	2.4751	3.6042	4.7167	5.8188	6.9137
2	3.7059	4.8859	6.0407	7.1799	8.3081	9.4280
3	6.1038	7.2902	8.4569	9.6099	10.7526	11.8972
4	8.5028	9.6925	10.8661	12.0277	13.1799	14.3244
5	10.9022	12.0939	13.2721	14.4396	15.5985	16.7502
6	13.3016	14.4949	15.6762	16.8481	18.0120	19.1692
7	15.7016	16.8596	18.0455	19.2545	20.4423	21.5829
8	18.1014	19.2581	20.4973	21.6281	22.9305	24.0056
9	20.5014	21.6281	22.8830	24.0056	25.3314	26.6086
10	22.9011	24.0056	25.2851	26.4667	27.6425	28.8130

Table 1.15: Roots λ_{nm} of associated Legendre Functions with respect to the degree for $\alpha = 75^\circ$

$\alpha = 50^\circ$

$n \setminus m$	0	1	2	3	4	5
1	2.2400	3.9207	5.4972	7.0226	8.5165	9.9886
2	5.8186	7.5556	9.2138	10.8231	12.3984	13.9483
3	9.4120	11.1692	12.8649	14.5176	16.1382	17.7337
4	13.0089	14.7765	16.4938	18.1735	19.8238	21.4501
5	16.6070	18.3809	20.1125	21.8107	23.4820	25.1309
6	20.2058	21.9840	23.7255	25.4374	27.1244	28.7905
7	23.8049	25.5862	27.3352	30.7567	32.4364	34.9130
8	27.4043	29.1879	30.9426	34.3822	36.0730	38.0728
9	31.0038	32.7892	34.5485	36.2855	38.0028	39.7028
10	34.6034	36.3902	38.1533	39.8957	41.6198	43.3275

Table 1.10: Roots λ_{nm} of associated Legendre Functions with respect to the degree for $\alpha = 50^\circ$

$\alpha = 55^\circ$

$n \setminus m$	0	1	2	3	4	5
1	1.9878	2.5249	4.3748	6.3821	7.7628	9.1252
2	5.2628	6.8647	8.3749	9.8213	11.2707	12.6982
3	8.5101	10.1107	11.6400	13.1527	14.5277	16.0457
4	11.7801	13.3897	14.9528	16.4928	18.0042	20.4358
5	15.0513	16.6659	18.2451	19.7866	21.3256	23.8358
6	18.3230	19.9412	21.5288	23.0916	24.6337	26.1583
7	21.5950	23.2157	24.8096	26.3810	27.9334	29.4693
8	24.8672	26.4899	28.0885	29.6667	31.2273	32.7720
9	28.1395	29.7637	31.3662	32.9499	34.5173	36.0701
10	31.4119	33.0373	34.6429	36.2312	37.8042	39.3635

Table 1.11: Roots λ_{nm} of associated Legendre Functions with respect to the degree for $\alpha = 55^\circ$

$\alpha = 60^\circ$

$n \setminus m$	0	1	2	3	4	5
1	1.7773	3.1957	4.5422	5.8532	7.1423	8.4162
2	4.7628	6.2195	7.6220	8.9908	10.3361	11.6638
3	7.7583	9.2288	10.6571	12.0577	13.4322	14.7914
4	10.7548	12.2338	13.6769	15.0942	16.4911	17.8716
5	13.7540	15.2369	16.6898	18.1598	19.5312	20.9272
6	16.7540	18.2400	19.6986	21.1361	22.5604	23.9682
7	19.7534	21.2405	22.7054	24.1519	25.5826	26.9998
8	22.7526	24.2416	25.7105	27.1626	28.6001	30.0250
9	25.7524	27.2426	28.7146	30.1712	31.6143	33.0455
10	28.7524	30.2433	31.7179	33.1793	34.6280	36.0625

Table 1.12: Roots λ_{nm} of associated Legendre Functions with respect to the degree for $\alpha = 60^\circ$

$\alpha = 80^\circ$

$\frac{n}{m}$	0	1	2	3	4	5
1	1.1956	2.2962	3.3747	4.4422	5.5028	6.5587
2	3.4414	4.5532	5.6486	6.7335	7.8113	8.8836
3	5.6900	6.8061	7.9092	9.0032	10.0905	11.1725
4	7.9394	9.0576	10.1652	11.2650	12.3585	13.4470
5	10.1890	11.3085	12.4191	13.5228	14.6207	15.7139
6	12.4387	13.5591	14.6719	15.7783	16.8796	17.9765
7	14.6885	15.8096	16.9239	18.0325	19.1364	20.2361
8	16.9384	18.0600	19.1755	20.2858	21.3917	22.4938
9	19.1883	20.3102	21.4267	22.5384	23.6461	24.7500
10	21.4382	22.5604	23.6777	24.7906	25.8996	27.0052

Table 1.16: Roots λ_{nm} of associated Legendre Functions with respect to the degree for $\alpha = 80^\circ$

$\alpha = 95^\circ$

$\frac{n}{m}$	0	1	2	3	4	5
1	0.9172	1.8764	2.8465	3.8221	4.8011	5.7825
2	2.8139	3.7677	4.7297	5.6969	6.6676	7.6411
3	4.7093	5.6611	6.6193	7.5820	8.5480	9.5167
4	6.6043	7.5551	8.5110	9.4709	10.4339	11.3994
5	8.4993	9.4494	10.4038	11.3618	12.3226	13.2857
6	10.3943	11.3438	12.2972	13.2536	14.2129	15.1742
7	12.2893	13.2381	14.1819	15.1464	16.0843	17.0652
8	14.1843	15.1323	16.0819	17.0396	17.9943	18.9562
9	16.0793	17.0274	17.9790	18.9396	19.8890	20.8468
10	17.9743	18.9221	19.8733	20.8267	21.7819	22.7389

Table 1.19: Roots λ_{nm} of associated Legendre Functions with respect to the degree for $\alpha = 95^\circ$

$\alpha = 85^\circ$

$\frac{n}{m}$	0	1	2	3	4	5
1	1.0922	2.1390	3.1748	4.2052	5.2322	6.2567
2	3.2078	4.2601	5.3042	6.3432	7.3786	8.4113
3	5.3248	6.3792	7.4271	8.4706	9.5107	10.5482
4	7.4421	8.4976	9.5178	10.5941	11.6373	12.6780
5	9.5596	10.6157	11.6674	12.7156	13.7610	14.8040
6	11.6771	12.7336	13.7884	14.8360	15.8830	16.9279
7	13.7946	14.8515	15.9050	16.9557	18.0041	19.0503
8	15.9121	16.9684	18.0234	19.0750	20.1244	21.1716
9	18.0296	19.0871	20.1410	21.1959	22.2441	23.2925
10	20.1474	21.2089	22.2590	23.3127	24.3636	25.4128

Table 1.17: Roots λ_{nm} of associated Legendre Functions with respect to the degree for $\alpha = 85^\circ$

$\alpha = 100^\circ$

$\frac{n}{m}$	0	1	2	3	4	5
1	0.8423	1.7659	2.7114	3.6670	4.6313	5.5904
2	2.6462	3.5698	4.4886	5.4084	6.3254	7.2378
3	4.4475	5.3664	6.2782	7.1909	8.1166	9.0396
4	6.2482	7.1549	8.0722	8.9974	9.9289	10.8653
5	8.0485	8.9540	9.8683	10.7897	11.7167	12.6481
6	9.8488	10.7534	11.6656	12.5841	13.5078	14.4360
7	11.6490	12.5529	13.4636	14.3799	15.3011	16.2263
8	13.4491	14.3523	15.2620	16.1767	17.0957	18.0187
9	15.2492	16.1523	17.0608	17.9740	18.8914	19.8124
10	17.0493	17.9521	18.8598	19.7719	20.6878	21.6072

Table 1.20: Roots λ_{nm} of associated Legendre Functions with respect to the degree for $\alpha = 100^\circ$

$\alpha = 90^\circ$

$\frac{n}{m}$	0	1	2	3	4	5
1	1.0000	2.0000	3.0000	4.0000	5.0000	6.0000
2	3.0000	4.0000	5.0000	6.0000	7.0000	8.0000
3	5.0000	6.0000	7.0000	8.0000	9.0000	10.0000
4	7.0000	8.0000	9.0000	10.0000	11.0000	12.0000
5	9.0000	10.0000	11.0000	12.0000	13.0000	14.0000
6	11.0000	12.0000	13.0000	14.0000	15.0000	16.0000
7	13.0000	14.0000	15.0000	16.0000	17.0000	18.0000
8	15.0000	16.0000	17.0000	18.0000	19.0000	20.0000
9	17.0000	18.0000	19.0000	20.0000	21.0000	22.0000
10	19.0000	20.0000	21.0000	22.0000	23.0000	24.0000

Table 1.18: Roots λ_{nm} of associated Legendre Functions with respect to the degree for $\alpha = 90^\circ$

$\alpha = 105^\circ$

$\frac{n}{m}$	0	1	2	3	4	5
1	0.7741	1.6667	2.5925	3.5347	4.4873	5.4471
2	2.4942	3.3710	4.2728	5.1902	6.1185	7.0551
3	4.2105	5.0811	5.9714	6.8753	7.7895	8.7117
4	5.9258	6.7932	7.6766	8.5720	9.4769	10.3894
5	7.6406	8.5061	9.3850	10.2745	11.1727	12.0781
6	9.3553	10.2194	11.0951	11.9804	12.8736	13.7734
7	11.0699	11.9330	12.8064	13.6883	14.5776	15.4731
8	12.7843	13.6467	14.5183	15.3977	16.2838	17.1758
9	14.4988	15.3606	16.2307	17.0957	17.9915	18.8806
10	16.2132	17.0745	17.9435	18.8190	19.7004	20.5870

Table 1.21: Roots λ_{nm} of associated Legendre Functions with respect to the degree for $\alpha = 105^\circ$

$\alpha = 125^\circ$

$n \setminus m$	0	1	2	3	4	5
1	0.5523	1.3583	2.2462	3.1734	4.1236	5.0886
2	2.0055	2.7753	3.6090	4.4826	5.3838	6.3053
3	3.4503	4.2050	5.0103	5.8511	6.7184	7.6065
4	4.8927	5.6393	6.4275	7.2468	8.0906	8.9543
5	6.3342	7.0757	7.8524	8.6567	9.4834	10.3289
6	7.7752	8.5133	9.2817	10.0749	10.8888	11.7201
7	9.2159	9.9515	10.7138	11.4985	12.3022	13.1223
8	10.6564	11.3901	12.1477	12.9256	13.7421	14.5321
9	12.0968	12.8290	13.5828	14.3553	15.1441	15.9475
10	13.5372	14.2681	15.0188	15.7868	16.5700	17.3669

Table 1.22: Roots λ_{nm} of associated Legendre Functions with respect to the degree for $\alpha = 125^\circ$

$\alpha = 130^\circ$

$n \setminus m$	0	1	2	3	4	5
1	0.5063	1.2988	2.1867	3.1192	4.0766	5.0492
2	1.9059	2.6571	3.4842	4.3583	5.2646	6.1943
3	3.2961	4.0298	4.8237	5.6599	6.5277	7.4201
4	4.6837	5.4077	6.1813	6.9923	7.8225	8.6965
5	6.0700	6.7880	7.5481	8.3412	9.1612	10.0036
6	7.4558	8.1697	8.9200	9.6999	10.5045	11.3301
7	8.8413	9.5522	10.2952	11.0649	11.8574	12.6696
8	10.2265	10.9351	11.6725	12.4342	13.2170	14.0182
9	11.6116	12.3185	13.0513	13.8064	14.5612	15.3153
10	12.9966	13.7020	14.4311	15.1809	15.9490	16.7334

Table 1.23: Roots λ_{nm} of associated Legendre Functions with respect to the degree for $\alpha = 130^\circ$

$\alpha = 135^\circ$

$n \setminus m$	0	1	2	3	4	5
1	0.4631	1.2451	2.1366	3.0771	4.0435	5.0243
2	1.8132	2.5890	3.3736	4.2534	5.1637	6.1018
3	3.1630	3.8865	4.6254	5.4241	6.2668	7.1497
4	4.5128	5.1838	5.9249	6.7245	7.6216	8.5597
5	5.8625	6.4734	7.2245	8.0555	8.9164	9.8172
6	7.2122	7.7532	8.5248	9.3255	10.1560	11.0744
7	8.5619	9.0340	9.9099	10.6688	11.4548	12.3243
8	9.9116	10.3144	11.2316	11.9819	12.7583	13.5548
9	11.2612	11.6062	12.5610	13.3025	14.0671	14.8531
10	12.6109	13.1783	13.8887	14.6238	15.3807	16.1572

Table 1.24: Roots λ_{nm} of associated Legendre Functions with respect to the degree for $\alpha = 135^\circ$

$\alpha = 110^\circ$

$n \setminus m$	0	1	2	3	4	5
1	0.7118	1.5775	2.4878	3.4203	4.3666	5.3223
2	2.1559	3.2006	4.0791	4.9788	5.8934	6.8190
3	3.5949	4.8314	5.6444	6.5759	7.4715	8.3780
4	5.0326	6.4647	7.3185	8.1886	9.0715	9.9648
5	6.4697	8.0991	8.9469	9.8091	10.6829	11.5665
6	7.9066	9.7342	10.5777	11.4340	12.3011	13.1773
7	9.3433	11.3696	12.2099	13.0617	13.9235	14.7938
8	10.7800	13.0053	13.8430	14.6914	15.5489	16.4144
9	12.2165	14.6411	15.4768	16.3223	17.1763	18.0379
10	13.6531	16.2770	17.1111	17.9543	18.8054	19.6635

Table 1.19: Roots λ_{nm} of associated Legendre Functions with respect to the degree for $\alpha = 110^\circ$

$\alpha = 115^\circ$

$n \setminus m$	0	1	2	3	4	5
1	0.6545	1.4969	2.3959	3.3229	4.2669	5.2226
2	2.2292	3.0456	3.9052	4.7917	5.6971	6.6165
3	3.7978	4.6038	5.4437	6.3073	7.1888	8.0842
4	5.3647	6.1651	6.9933	7.8423	8.7078	9.5865
5	6.9309	7.7279	8.5484	9.3873	10.2412	11.1075
6	8.4968	9.2915	10.1064	10.9379	11.7831	12.6400
7	10.0625	10.8555	11.6663	12.4921	13.3305	14.1799
8	11.6281	12.4198	13.2275	14.0487	14.8817	15.7249
9	13.1935	13.9843	14.7894	15.6070	16.4355	17.2736
10	14.7590	15.5489	16.3520	17.1666	17.9913	18.8252

Table 1.20: Roots λ_{nm} of associated Legendre Functions with respect to the degree for $\alpha = 115^\circ$

$\alpha = 120^\circ$

$n \setminus m$	0	1	2	3	4	5
1	0.6015	1.4241	2.3157	3.2410	4.1865	5.1455
2	2.1129	2.9043	3.7490	4.6268	5.5277	6.4458
3	3.6169	4.3957	5.2164	6.0664	6.9384	7.8276
4	5.1190	5.8910	6.6973	7.5294	8.3817	9.2505
5	6.6202	7.3880	8.1843	9.0044	9.8426	10.6961
6	8.1210	8.8860	9.6760	10.4864	11.3138	12.1577
7	9.6216	10.3845	11.1694	11.9728	12.7919	13.6245
8	11.1221	11.8833	12.6643	13.4622	14.2745	15.0995
9	12.6224	13.3824	14.1603	14.9536	15.7604	16.5791
10	14.1227	14.8817	15.6570	16.4466	17.2488	18.0622

Table 1.21: Roots λ_{nm} of associated Legendre Functions with respect to the degree for $\alpha = 120^\circ$

$\alpha = 155^\circ$

$n \backslash m$	0	1	2	3	4	5
1	0.3101	1.0825	2.0203	3.0046	4.0010	5.0002
2	1.4951	2.2045	3.0744	4.0235	5.0066	6.0017
3	2.6670	3.3435	4.1596	5.0652	6.0232	7.0073
4	3.8342	4.4909	5.2672	6.1311	7.0568	8.0216
5	4.9994	5.6428	6.3957	7.2180	8.1102	9.0495
6	6.1633	6.7973	7.5220	8.3212	9.1624	10.0939
7	7.3265	7.9535	8.6810	9.4366	10.2707	11.1550
8	8.4898	9.1099	9.8016	10.5609	11.3715	12.2313
9	9.6531	10.2658	10.9318	11.6859	12.7652	13.3206
10	10.8164	11.4278	12.1014	12.8281	13.6026	14.4207

Table 1.28: Roots λ_{nm} of associated Legendre Functions with respect to the degree for $\alpha = 155^\circ$

$\alpha = 160^\circ$

$n \backslash m$	0	1	2	3	4	5
1	0.2745	1.0514	2.0092	3.0014	4.0002	5.0000
2	1.4747	2.1396	3.0352	4.0019	5.0014	6.0002
3	2.5615	3.2405	4.0852	5.0274	6.0057	7.0004
4	3.6224	4.3496	5.1631	6.0542	7.0161	8.0106
5	4.6829	5.4635	6.2328	7.1000	8.0377	9.0235
6	5.7511	6.5804	7.3236	8.1592	9.0668	10.0235
7	6.8285	7.6993	8.4217	9.2301	10.1100	11.0448
8	7.9153	8.8194	9.5252	10.3104	11.1646	12.0761
9	9.0117	9.9406	10.6327	11.3983	12.2293	13.1176
10	10.1179	11.0624	11.7431	12.4920	13.3027	14.1691

Table 1.29: Roots λ_{nm} of associated Legendre Functions with respect to the degree for $\alpha = 160^\circ$

$\alpha = 165^\circ$

$n \backslash m$	0	1	2	3	4	5
1	0.2387	1.0316	2.0031	3.0003	4.0000	5.0000
2	1.3560	2.0844	3.0137	4.0017	5.0002	6.0000
3	2.4599	3.1499	4.0349	5.0058	6.0008	7.0001
4	3.5588	4.2231	5.0674	6.0147	7.0024	8.0003
5	4.6550	5.3011	6.1103	7.0302	8.0062	9.0010
6	5.7498	6.3822	7.1621	8.0534	9.0133	10.0026
7	6.8436	7.4656	8.2210	9.0847	10.0250	11.0057
8	7.9368	8.5505	9.2855	10.1237	11.0425	12.0113
9	9.0295	9.6365	10.3544	11.1697	12.0663	13.0204
10	10.1219	10.7233	11.4267	12.2217	13.0966	14.0339

Table 1.30: Roots λ_{nm} of associated Legendre Functions with respect to the degree for $\alpha = 165^\circ$

$\alpha = 140^\circ$

$n \backslash m$	0	1	2	3	4	5
1	0.4223	1.1968	2.0954	3.0462	4.0220	5.0103
2	1.7266	2.4501	3.2775	4.1677	5.0985	6.0960
3	3.0197	3.7200	4.5045	5.3477	6.2341	7.1532
4	4.3093	4.9867	5.7537	6.5632	7.4140	8.2980
5	5.5974	6.2765	7.0148	7.9599	8.6231	9.4783
6	6.8848	7.5281	8.2829	9.0496	9.8515	10.6634
7	8.1717	8.8408	9.5554	10.3075	11.0928	11.9059
8	9.4562	10.1242	10.8388	11.5720	12.3452	13.5402
9	10.7446	11.4681	12.1083	12.8403	13.8002	14.8849
10	12.0309	12.8242	13.3673	14.1117	14.8821	15.6355

Table 1.25: Roots λ_{nm} of associated Legendre Functions with respect to the degree for $\alpha = 140^\circ$

$\alpha = 145^\circ$

$n \backslash m$	0	1	2	3	4	5
1	0.3834	1.1537	2.0656	3.0250	4.0096	5.0036
2	1.6222	2.4029	3.2692	4.1079	5.0499	6.0331
3	2.8685	3.5931	4.4711	5.2274	6.1330	7.0737
4	4.1109	4.8142	5.5704	6.3890	7.2560	8.1615
5	5.3850	6.0487	6.7831	7.5733	8.4091	9.2827
6	6.6283	7.2853	8.0037	8.7723	9.5831	10.4301
7	7.8711	8.5231	9.2294	10.7720	11.5967	12.7777
8	9.1135	9.7618	10.4586	11.1970	12.7777	14.1693
9	10.3557	11.0010	11.6903	12.4176	13.1786	14.9694
10	11.5977	12.2406	12.9237	13.6419	14.3915	15.1693

Table 1.26: Roots λ_{nm} of associated Legendre Functions with respect to the degree for $\alpha = 145^\circ$

$\alpha = 150^\circ$

$n \backslash m$	0	1	2	3	4	5
1	0.3462	1.1156	2.0378	3.0117	4.0035	5.0010
2	1.5683	2.2780	3.1274	4.0536	5.0207	6.0074
3	2.7777	3.4580	4.2556	5.1328	6.0633	7.0277
4	3.9829	4.6458	5.4078	6.2438	7.1361	8.0702
5	5.1862	5.8378	6.5713	7.3780	8.2361	9.1384
6	6.3884	7.0321	7.7498	8.5282	9.3577	10.2306
7	7.5901	8.2279	8.9312	9.6896	10.4955	11.3428
8	8.7913	9.4246	10.1166	10.8590	11.6454	12.4708
9	9.9923	10.6220	11.3049	12.0341	12.8042	13.6111
10	11.1930	11.8199	12.4953	13.2135	13.9699	14.7610

Table 1.27: Roots λ_{nm} of associated Legendre Functions with respect to the degree for $\alpha = 150^\circ$

$\alpha = 170^\circ$

$n \setminus m$	0	1	2	3	4	5
1	0.2012	1.0145	2.0007	3.0000	4.0000	5.0000
2	1.2869	2.0405	3.0031	4.0002	5.0000	6.0000
3	3.4277	3.0748	4.0086	5.0018	6.0000	7.0000
4	4.4928	4.1149	5.0181	6.0018	7.0001	8.0000
5	5.5654	5.1592	6.0324	7.0040	8.0004	9.0000
6	6.6189	6.2065	7.0515	8.0080	9.0008	10.0001
7	7.6806	7.2560	8.0753	9.0142	10.0018	11.0002
8	8.7418	8.3072	9.1035	10.0233	11.0035	12.0004
9	9.8027	9.3596	10.1355	11.0355	12.0062	13.0008
10	10.8635	10.4130	11.1708	12.0511	13.0104	14.0015

Table 1.31: Roots λ_{nm} of associated Legendre Functions with respect to the degree for $\alpha = 170^\circ$

$\alpha = 5^\circ$

$n \setminus m$	0	1	2	3	4	5
1	43.4110	20.6155	34.5252	47.6795	60.4841	73.0795
2	79.8943	60.5987	76.3571	91.3658	105.8950	120.0848
3	116.0804	97.3220	113.7487	129.5273	144.8433	159.8083
4	152.1791	133.6431	150.4266	166.6513	182.4505	197.9123
5	188.2402	169.8259	186.8330	203.3520	219.4830	235.2963
6	224.2817	205.9442	223.1051	239.8337	256.2078	272.2847
7	260.3118	242.0270	259.3002	276.1860	292.7466	309.0292
8	296.3346	278.0883	295.4473	312.4554	329.1636	345.6116
9	332.3525	314.1356	331.5621	348.6682	365.4963	382.0802
10	368.3669	350.1730	367.6542	384.8407	401.7682	418.4659

Table 2.1: Roots λ_{nm} of the derivative of the associated Legendre Functions with respect to the degree for $\alpha = 5^\circ$

$\alpha = 10^\circ$

$n \setminus m$	0	1	2	3	4	5
1	21.4598	10.0835	17.0520	23.6459	30.0663	36.3829
2	39.6995	30.0567	37.9444	45.4600	52.7375	59.8465
3	57.7918	48.4155	56.6349	64.5326	72.2007	79.6947
4	75.8408	66.5748	74.9712	83.0903	90.9983	98.7388
5	93.8711	84.6655	93.1729	101.4380	109.5106	117.4256
6	111.8917	102.7242	111.3079	119.6770	127.8703	135.9161
7	129.9066	120.7653	129.4047	137.8519	146.1376	154.2855
8	147.9180	138.7957	147.4777	155.9855	164.3446	172.5745
9	165.9268	156.8192	165.5346	174.0911	182.5097	190.8070
10	183.9340	174.8378	183.5803	192.1767	200.6445	208.9984

Table 2.2: Roots λ_{nm} of the derivative of the associated Legendre Functions with respect to the degree for $\alpha = 10^\circ$

$\alpha = 15^\circ$

$n \setminus m$	0	1	2	3	4	5
1	14.1446	6.5842	11.2452	15.6600	19.9605	24.1926
2	28.3023	19.8793	25.1472	30.1702	35.0364	39.7915
3	38.3650	32.1154	37.6016	42.8762	47.9996	53.0083
4	50.3952	44.2202	49.8230	55.2433	60.5246	65.6956
5	62.4482	56.2799	61.9557	67.4721	72.8618	78.1477
6	74.4987	68.3185	74.0045	79.6294	85.0985	90.4705
7	86.4486	80.3456	86.1083	91.7444	97.2744	102.7136
8	98.4466	92.3656	98.1563	103.8324	109.4106	114.9038
9	110.4518	104.3810	110.1937	115.9019	121.5192	127.0568
10	122.4565	116.3932	122.2230	127.9582	133.6080	139.1628

Table 2.3: Roots λ_{nm} of the derivative of the associated Legendre Functions with respect to the degree for $\alpha = 15^\circ$

$\alpha = 20^\circ$

n/m	0	1	2	3	4	5
1	5.7930	2.6347	4.6872	6.6463	8.5616	10.4504
2	10.9358	8.2626	12.7595	16.8870	21.9877	27.5911
3	16.1619	13.4957	18.1640	23.4475	29.3996	35.5976
4	21.3171	18.6787	25.7437	31.0158	37.9976	45.5976
5	26.4675	23.8444	31.6917	38.5811	46.6545	55.5976
6	31.6156	29.0020	37.4686	44.2445	54.2334	64.5976
7	36.7622	34.1552	42.2445	49.4140	61.8389	73.5976
8	41.9079	39.3057	47.0172	54.5784	69.4441	82.5976
9	47.0530	44.4545	51.7920	60.7371	77.0493	91.5976
10	52.1977	49.6020	56.9616	66.8900	84.6545	100.5976

Table 2.7: Roots λ_{nm} of the derivative of the associated Legendre Functions with respect to the degree for $\alpha = 35^\circ$

$\alpha = 40^\circ$

n/m	0	1	2	3	4	5
1	5.0120	2.2754	4.0905	5.8278	7.5284	9.2069
2	9.5619	7.1770	9.1928	11.1301	13.0168	14.8677
3	14.0813	11.7522	13.8381	15.8561	17.8257	19.7585
4	18.5916	16.2857	18.4088	20.4735	22.4938	24.4787
5	23.0979	20.8047	22.9512	25.0467	27.1016	29.1234
6	27.6022	25.3170	27.4795	29.5969	31.6773	33.7266
7	32.1054	29.8257	31.9959	34.1337	36.2335	38.3044
8	36.6078	34.3321	36.5152	38.6618	40.7771	42.8651
9	41.1096	38.8370	41.0272	43.1840	45.3118	47.4141
10	45.6111	43.3409	45.5368	47.7020	49.8401	51.9543

Table 2.8: Roots λ_{nm} of the derivative of the associated Legendre Functions with respect to the degree for $\alpha = 40^\circ$

$\alpha = 45^\circ$

n/m	0	1	2	3	4	5
1	4.4053	2.0000	3.6332	5.2014	6.7388	8.2575
2	8.4471	6.3339	8.1373	9.8747	11.5696	13.2344
3	12.4633	10.3970	12.2592	14.0645	15.8291	17.5628
4	16.4719	14.4251	16.3186	18.1631	19.9702	21.7477
5	20.4773	18.4410	20.3541	22.2245	24.0608	25.8692
6	24.4809	22.4514	24.3779	26.2667	28.1243	29.9559
7	28.4835	26.4586	28.3950	30.2975	32.1715	34.0271
8	32.4855	30.4640	32.4079	34.3211	36.2080	38.0721
9	36.4871	34.4681	36.4179	38.3397	40.2372	42.1132
10	40.4884	38.4714	40.4260	42.3548	44.2610	46.1470

Table 2.9: Roots λ_{nm} of the derivative of the associated Legendre Functions with respect to the degree for $\alpha = 45^\circ$

$\alpha = 20^\circ$

n/m	0	1	2	3	4	5
1	10.4885	4.8432	8.3553	14.9332	18.1296	21.5825
2	19.6044	14.7931	18.7541	26.1995	29.7825	34.9790
3	28.6492	23.9669	32.0545	35.9090	39.6790	49.1852
4	37.6729	33.0440	37.2516	41.3248	45.2956	58.5151
5	46.6876	42.0880	46.3493	50.4933	54.3439	67.5558
6	55.6975	51.1164	55.4147	59.6090	63.7182	76.9348
7	64.7048	60.1363	64.4616	68.6938	72.8477	86.0749
8	73.7103	69.1510	73.4869	77.7585	81.9480	94.8875
9	82.7145	78.1624	82.5245	86.8097	91.0283	104.2803
10	91.7180	87.1714	91.5467	95.8511	100.0933	113.6631

Table 2.4: Roots λ_{nm} of the derivative of the associated Legendre Functions with respect to the degree for $\alpha = 20^\circ$

$\alpha = 25^\circ$

n/m	0	1	2	3	4	5
1	8.2960	3.8056	6.6322	9.3179	11.9378	14.5182
2	15.5864	11.7434	14.9226	17.9607	20.9086	23.7924
3	22.8213	19.0790	22.3837	25.5668	28.6628	31.6928
4	30.0398	26.3392	29.7110	32.9778	36.1647	39.2881
5	37.2513	33.5735	36.9872	40.3093	43.5585	46.7481
6	44.4591	40.7957	44.2383	47.5997	50.8946	54.1336
7	51.6647	48.0113	51.4749	54.8658	58.1957	61.4734
8	58.8690	55.2228	58.7025	62.1164	65.4740	68.7827
9	66.0723	62.4316	65.9240	69.3563	72.7364	76.0707
10	73.2750	69.6387	73.1413	76.5887	79.9874	83.3431

Table 2.5: Roots λ_{nm} of the derivative of the associated Legendre Functions with respect to the degree for $\alpha = 25^\circ$

$\alpha = 30^\circ$

n/m	0	1	2	3	4	5
1	6.8354	3.1196	5.4928	7.7524	9.9589	12.1334
2	12.9083	9.7121	12.3720	14.9180	17.3910	19.6121
3	18.9314	15.8215	18.8930	21.2461	23.8390	26.2784
4	24.9514	21.8702	24.6958	27.4166	30.0829	32.6979
5	30.9606	27.8797	30.7874	33.5229	36.2395	38.9079
6	36.9699	33.8158	36.7886	39.5959	42.3494	44.9579
7	42.9715	39.7323	42.8182	45.6493	48.4311	51.1708
8	48.9747	45.6318	48.8405	51.8902	54.4841	57.2551
9	54.9797	51.5168	54.9848	58.1448	60.5448	63.3248
10	60.9796	57.3905	60.8718	64.3485	66.5860	69.3886

Table 2.6: Roots λ_{nm} of the derivative of the associated Legendre Functions with respect to the degree for $\alpha = 30^\circ$

$\alpha = 65^\circ$

$n \setminus m$	0	1	2	3	4	5
1	2.9178	1.3513	2.5608	3.7405	4.9056	6.0618
2	5.7062	4.2688	5.5614	6.8231	8.0649	9.2924
3	8.4829	6.3876	8.3876	9.6813	10.9559	12.2160
4	11.2562	9.8497	11.1846	12.4966	13.7912	15.0719
5	14.0279	12.6264	13.9704	15.2945	16.6028	17.8979
6	16.7988	15.4004	16.7508	18.0835	19.4017	20.7077
7	19.5693	18.1730	19.5280	20.8671	22.1929	23.5074
8	22.3394	20.9447	22.3032	23.6473	24.9792	26.3004
9	25.1094	23.7159	25.0771	26.4252	27.7620	29.0888
10	27.8792	26.4867	27.8501	29.2015	30.5424	31.8738

Table 2.13: Roots λ'_{nm} of the derivative of the associated Legendre Functions with respect to the degree for $\alpha = 65^\circ$

$\alpha = 70^\circ$

$n \setminus m$	0	1	2	3	4	5
1	2.6803	1.2543	2.4025	3.5274	4.6410	5.7476
2	5.2665	3.9396	5.1535	6.3431	7.5172	8.6801
3	7.8438	6.5340	7.7694	8.9841	10.1839	11.3723
4	10.4184	9.1156	10.3625	11.5915	12.8069	14.0115
5	12.9917	11.6328	12.9469	14.1854	15.4114	16.6273
6	15.5645	14.2680	15.5270	16.7722	18.0060	19.2304
7	18.1369	16.8421	18.1046	19.3548	20.5947	21.8256
8	20.7090	19.4155	20.6808	21.9349	23.1794	24.4156
9	23.2810	21.9884	23.2558	24.5131	25.7614	27.0020
10	25.8529	24.5610	25.8302	27.0900	28.3415	29.5857

Table 2.14: Roots λ'_{nm} of the derivative of the associated Legendre Functions with respect to the degree for $\alpha = 70^\circ$

$\alpha = 75^\circ$

$n \setminus m$	0	1	2	3	4	5
1	4.8859	1.1731	2.2712	3.3523	4.4250	5.4925
2	7.2902	3.6556	4.8026	5.9318	7.0495	8.1590
3	9.6925	6.0724	7.2354	8.3831	9.5198	10.6481
4	12.0939	8.4799	9.6513	10.8096	11.9578	13.0961
5	14.4949	10.8842	12.0609	13.2262	14.3822	15.5308
6	16.8956	13.2870	14.4673	15.6274	16.7993	17.9571
7	19.2961	15.6889	16.8719	18.0457	19.1419	20.3716
8	21.6955	18.0904	19.2783	20.3240	21.4336	22.7843
9	24.0959	20.4923	21.5100	22.5611	23.6356	25.1963
10	26.4963	22.8943	22.7400	24.8002	26.4359	27.6034

Table 2.15: Roots λ'_{nm} of the derivative of the associated Legendre Functions with respect to the degree for $\alpha = 75^\circ$

$\alpha = 50^\circ$

$n \setminus m$	0	1	2	3	4	5
1	3.9207	1.7834	3.2740	4.7102	6.1204	7.5149
2	7.5556	5.6607	7.2956	8.9760	10.4188	11.9373
3	11.1892	9.3136	10.8979	12.6345	14.2369	15.8132
4	14.7765	12.9371	14.6477	16.3173	17.9554	19.5686
5	18.3809	16.5505	18.2776	19.9688	21.6314	23.2705
6	21.9840	20.1592	21.8916	23.6042	25.2847	26.9434
7	25.5862	23.7653	25.5119	27.2301	28.9243	30.5981
8	29.1879	27.3698	29.1227	30.8499	32.5550	34.2410
9	32.7892	30.9732	32.7311	34.4655	36.1794	37.8754
10	36.3902	34.5760	36.3379	38.0782	39.7994	41.5038

Table 2.10: Roots λ'_{nm} of the derivative of the associated Legendre Functions with respect to the degree for $\alpha = 50^\circ$

$\alpha = 55^\circ$

$n \setminus m$	0	1	2	3	4	5
1	3.5249	1.6096	2.9863	4.3177	5.6274	6.9240
2	6.8257	5.1110	6.6095	8.0617	9.4840	10.8851
3	10.1107	8.4279	9.9677	11.4677	12.9390	14.3806
4	13.3895	11.7203	13.2820	14.8095	16.3107	17.7911
5	16.6659	15.0042	16.5797	18.1252	19.6469	21.1490
6	19.9412	18.2841	19.8691	21.4275	22.9642	24.4825
7	23.2157	21.5619	23.1538	24.7218	26.2659	27.8010
8	26.4899	24.8384	26.4355	28.0111	29.5683	31.1094
9	29.7637	28.1140	29.7153	31.2969	32.8614	34.4109
10	33.0373	31.3891	32.9936	34.5802	36.1508	37.7073

Table 2.11: Roots λ'_{nm} of the derivative of the associated Legendre Functions with respect to the degree for $\alpha = 55^\circ$

$\alpha = 60^\circ$

$n \setminus m$	0	1	2	3	4	5
1	3.1957	1.4680	2.7526	4.0000	5.2295	6.4480
2	6.2195	4.6542	6.0404	7.3884	8.7117	10.0174
3	9.2288	7.6905	9.1109	10.4984	11.8623	13.2082
4	12.2338	10.7067	12.1452	13.5554	14.9439	16.3153
5	15.2369	13.7160	15.1658	16.5909	17.9963	19.3856
6	18.2390	16.7219	18.1795	19.6152	21.0330	22.4357
7	21.2405	19.7261	21.1894	22.6331	24.0603	25.4734
8	24.2416	22.7292	24.1968	25.6467	27.0814	28.5029
9	27.2426	25.7316	27.2026	28.6575	30.0982	31.5266
10	30.2433	28.7335	30.2073	31.6662	33.1120	34.5461

Table 2.12: Roots λ'_{nm} of the derivative of the associated Legendre Functions with respect to the degree for $\alpha = 60^\circ$

$\alpha = 80^\circ$

$n \setminus m$	0	1	2	3	4	5
1	2.2962	1.1050	2.1626	3.2090	4.2498	5.2873
2	4.5532	3.4083	4.4983	5.5767	6.6475	7.7128
3	6.8061	5.6693	6.7700	7.8605	8.9438	10.0216
4	9.0576	7.9243	9.0320	10.1280	11.2189	12.3046
5	11.3085	10.1771	11.2868	12.3889	13.4850	14.5761
6	13.5591	12.4289	13.5410	14.6463	15.7462	16.8414
7	15.8096	14.6802	15.7940	16.9018	18.0045	19.1030
8	18.0600	16.9311	18.0463	19.1559	20.2610	21.3620
9	20.3102	19.1819	20.2980	21.4092	22.5161	23.6193
10	22.5604	21.4325	22.5494	23.6619	24.7703	25.8752

Table 2.16: Roots λ_{nm} of the derivative of the associated Legendre Functions with respect to the degree for $\alpha = 80^\circ$

$\alpha = 85^\circ$

$n \setminus m$	0	1	2	3	4	5
1	2.1390	1.0478	2.0730	3.0927	4.1096	5.1248
2	4.2601	3.1914	4.2328	5.2685	6.3004	7.3295
3	6.3792	5.3145	6.3612	7.4030	8.4411	9.4765
4	8.4976	7.4346	8.4841	9.5293	10.5712	11.6105
5	10.6157	9.5537	10.6049	11.6523	12.6968	13.7388
6	12.7336	11.6722	12.7246	13.7737	14.8200	15.8641
7	14.8515	13.7905	14.8438	15.8940	16.9418	17.9875
8	16.9693	15.9086	16.9625	18.0138	19.0627	20.1056
9	19.0871	18.0266	19.0811	20.1330	21.1829	22.2308
10	21.2049	20.1446	21.1994	22.2520	23.3026	24.3514

Table 2.17: Roots λ_{nm} of the derivative of the associated Legendre Functions with respect to the degree for $\alpha = 85^\circ$

$\alpha = 90^\circ$

$n \setminus m$	0	1	2	3	4	5
1	2.0000	1.0000	2.0000	3.0000	4.0000	5.0000
2	4.0000	3.0000	4.0000	5.0000	6.0000	7.0000
3	6.0000	5.0000	6.0000	7.0000	8.0000	9.0000
4	8.0000	7.0000	8.0000	9.0000	10.0000	11.0000
5	10.0000	9.0000	10.0000	11.0000	12.0000	13.0000
6	12.0000	11.0000	12.0000	13.0000	14.0000	15.0000
7	14.0000	13.0000	14.0000	15.0000	16.0000	17.0000
8	16.0000	15.0000	16.0000	17.0000	18.0000	19.0000
9	18.0000	17.0000	18.0000	19.0000	20.0000	21.0000
10	20.0000	19.0000	20.0000	21.0000	22.0000	23.0000

Table 2.18: Roots λ_{nm} of the derivative of the associated Legendre Functions with respect to the degree for $\alpha = 90^\circ$

$\alpha = 95^\circ$

$n \setminus m$	0	1	2	3	4	5
1	1.8764	0.9602	1.9416	2.9283	3.9178	4.9089
2	3.7677	2.8303	3.7951	4.7658	5.7402	6.7173
3	5.6611	4.7195	5.6790	6.6436	7.6117	8.5825
4	7.5551	6.6118	7.5685	8.5296	9.4939	10.4608
5	9.4494	8.5052	9.4602	10.4189	11.3806	12.3448
6	11.3438	10.3990	11.3528	12.3059	13.2698	14.2319
7	13.2383	12.2931	13.2460	14.2019	15.1604	16.1209
8	15.1328	14.1874	15.1396	16.0946	17.0519	18.0112
9	17.0274	16.0877	17.0335	18.0046	19.9441	20.9023
10	18.9221	17.9762	18.9275	19.8811	20.8367	21.7941

Table 2.19: Roots λ_{nm} of the derivative of the associated Legendre Functions with respect to the degree for $\alpha = 95^\circ$

$\alpha = 100^\circ$

$n \setminus m$	0	1	2	3	4	5
1	1.7659	0.9276	1.8962	2.8756	3.8604	4.8485
2	3.5592	2.6792	3.6146	4.5617	5.5164	6.4764
3	5.3564	4.4682	5.3927	6.3273	7.2691	8.2165
4	7.1549	6.2632	7.1821	8.1096	9.0438	9.9832
5	8.9540	8.0604	8.9758	9.8987	10.8277	11.7616
6	10.7534	9.8586	10.7716	11.6912	12.6164	13.5462
7	12.5529	11.6573	12.5685	13.4858	14.4080	15.3346
8	14.3525	13.4564	14.3663	15.2816	16.2015	17.1255
9	16.1523	15.2556	16.1645	17.0783	17.9964	18.9183
10	17.9521	17.0550	17.9631	18.8756	19.7922	20.7123

Table 2.20: Roots λ_{nm} of the derivative of the associated Legendre Functions with respect to the degree for $\alpha = 100^\circ$

$\alpha = 105^\circ$

$n \setminus m$	0	1	2	3	4	5
1	1.6667	0.9013	1.8629	2.8404	3.8259	4.8162
2	3.3710	2.5444	3.4556	4.3846	5.3252	6.2740
3	5.0811	4.2419	5.1364	6.0463	6.9671	7.8953
4	6.7932	5.9487	6.8345	7.7336	8.6427	9.5598
5	8.5061	7.6587	8.5392	9.4312	10.3324	11.2411
6	10.2194	9.3702	10.2471	11.1341	12.0295	12.9319
7	11.9330	11.0825	11.9567	12.8401	13.7311	14.6286
8	13.6467	12.7954	13.6675	14.5480	15.4355	16.3291
9	15.3606	14.5085	15.3792	16.2573	17.1419	18.0324
10	17.0745	16.2220	17.0913	17.9675	18.8498	19.7376

Table 2.21: Roots λ_{nm} of the derivative of the associated Legendre Functions with respect to the degree for $\alpha = 105^\circ$

$\alpha = 110^\circ$

$n \setminus m$	0	1	2	3	4	5
1	1.5775	0.8809	1.8407	2.8215	3.8124	4.8094
2	3.2006	2.4240	3.3161	4.2326	5.1643	6.1084
3	6.1844	4.0275	4.9067	5.7966	6.7016	7.6180
4	8.9917	5.9937	7.3962	8.2851	9.1848	10.0948
5	8.0991	7.9937	9.1442	10.0098	10.8673	11.7611
6	9.7362	8.9268	10.6307	11.5008	12.3805	13.8929
7	11.3696	10.5605	11.7718	12.8577	14.1059	15.6106
8	13.0053	12.1949	13.0332	14.8577	16.3685	17.5819
9	14.6411	13.8298	14.5663	16.5134	18.1438	19.5559
10	16.2770	15.4650	16.2997	17.1438	19.9662	21.5329

Table 2.22: Roots λ'_{nm} of the derivative of the associated Legendre Functions with respect to the degree for $\alpha = 110^\circ$

$\alpha = 115^\circ$

$n \setminus m$	0	1	2	3	4	5
1	1.4969	0.8660	1.8291	2.8180	3.8180	4.8248
2	3.0456	2.3165	3.1948	4.1044	5.0315	5.9755
3	4.6038	3.8525	4.7007	5.5757	6.4699	7.3891
4	6.1651	5.4046	6.2374	7.0933	7.9667	8.8611
5	7.7279	6.9623	7.7858	8.6292	9.4887	10.3612
6	9.2915	8.5227	9.3357	10.1745	11.0238	11.8853
7	10.8555	10.0845	10.8969	11.7252	12.5667	13.4197
8	12.4198	11.6472	12.4561	13.2793	14.1148	14.9609
9	13.9843	13.2105	14.0166	14.8358	15.6663	16.5068
10	15.5489	14.7742	15.5780	16.3939	17.2204	18.0562

Table 2.23: Roots λ'_{nm} of the derivative of the associated Legendre Functions with respect to the degree for $\alpha = 115^\circ$

$\alpha = 120^\circ$

$n \setminus m$	0	1	2	3	4	5
1	1.4241	0.8563	1.8274	2.8275	3.8397	4.8571
2	3.2904	2.2209	3.0908	4.0000	4.9352	5.8896
3	4.3957	3.6847	4.5164	5.3816	6.2708	7.1787
4	5.8910	5.1684	5.9808	6.8219	7.6848	8.5651
5	7.3880	6.6591	7.4598	8.2853	9.1307	9.9923
6	8.8860	8.1531	8.9458	9.7605	10.5932	11.4411
7	10.3845	9.6489	10.4358	11.2424	12.0655	12.9027
8	11.8833	11.1458	11.9283	12.7286	13.5442	14.3729
9	13.3824	12.6434	13.4225	14.2178	15.0272	15.8490
10	14.8817	14.1415	14.9178	15.7090	16.5133	17.3293

Table 2.24: Roots λ'_{nm} of the derivative of the associated Legendre Functions with respect to the degree for $\alpha = 120^\circ$

$\alpha = 125^\circ$

$n \setminus m$	0	1	2	3	4	5
1	1.3583	0.8517	2.8191	3.8729	4.8986	5.8825
2	2.7753	2.1364	3.9206	4.8696	5.8425	6.8171
3	4.2050	3.5325	4.9526	5.7437	6.7019	7.6572
4	5.6393	4.9527	5.7489	6.5799	7.4377	8.3171
5	7.0757	6.3814	7.1631	7.9751	8.8112	9.6672
6	8.5133	7.8141	8.5860	9.8047	10.2052	11.0443
7	9.9515	9.2490	10.0139	10.8026	11.6114	12.4375
8	11.3901	10.6852	11.4447	12.2258	13.0254	13.8411
9	12.8280	12.1223	12.8776	14.4447	14.4447	15.2519
10	14.2661	13.5600	14.3119	15.8679	16.8680	17.6680

Table 2.25: Roots λ'_{nm} of the derivative of the associated Legendre Functions with respect to the degree for $\alpha = 125^\circ$

$\alpha = 130^\circ$

$n \setminus m$	0	1	2	3	4	5
1	1.2988	0.8520	1.8512	2.8791	3.9112	4.9389
2	2.6571	2.0628	2.9355	3.8684	4.8410	5.8408
3	4.0298	3.3948	4.2086	4.9276	5.9740	6.9955
4	5.4027	4.7596	5.5393	5.9665	6.2558	7.2588
5	6.7800	6.1496	6.9252	6.8984	6.8268	7.8393
6	8.1600	7.5496	8.3232	8.4021	8.4021	8.8372
7	9.5427	8.9496	9.6572	10.4021	11.2715	12.1410
8	10.9255	10.3496	11.0608	11.7665	12.5542	13.3616
9	12.3083	11.7496	12.3768	13.1352	13.9141	14.7111
10	13.6911	13.1496	13.7546	14.5069	15.2784	16.0668

Table 2.26: Roots λ'_{nm} of the derivative of the associated Legendre Functions with respect to the degree for $\alpha = 130^\circ$

$\alpha = 135^\circ$

$n \setminus m$	0	1	2	3	4	5
1	1.2051	0.8572	1.8713	2.9130	3.9466	4.9694
2	2.5490	2.0000	2.8868	3.8424	4.8506	5.8776
3	3.8938	3.2709	4.0850	4.9612	5.8850	6.8477
4	5.2386	4.5756	5.3532	6.1822	7.0520	7.9560
5	6.5834	5.8823	6.6486	7.4486	8.2842	9.1501
6	7.9282	7.1855	7.9570	8.7371	9.5492	10.3887
7	9.2730	8.4883	9.2727	10.0382	10.8329	11.6527
8	10.6178	9.7894	10.5928	11.3472	12.1283	12.9327
9	11.9626	11.0904	11.9159	12.6615	13.4316	14.2236
10	13.3074	12.3910	13.2410	13.9794	14.7406	15.5222

Table 2.27: Roots λ'_{nm} of the derivative of the associated Legendre Functions with respect to the degree for $\alpha = 135^\circ$

$\alpha = 140^\circ$

$n \backslash m$	0	1	2	3	4	5
1	1.1968	0.8672	1.9019	2.9449	3.9728	4.9874
2	2.4501	1.9487	2.8602	3.8556	4.8897	5.9300
3	3.7200	3.1605	3.9837	4.8851	5.8472	6.8343
4	4.9967	4.4119	5.1893	6.0303	6.9237	7.8631
5	6.2765	5.6781	6.4285	7.2328	8.0815	8.9695
6	7.5583	6.9512	7.6840	8.4635	9.2822	10.1352
7	8.8426	8.2261	8.9483	9.7099	10.5069	11.3347
8	10.1292	9.5034	10.2161	10.9662	11.7464	12.5550
9	11.4182	10.7881	11.4715	12.2288	12.9957	13.7890
10	12.6924	12.0699	12.7674	13.4960	14.2520	15.0327

Table 2.28: Roots λ'_{nm} of the derivative of the associated Legendre Functions with respect to the degree for $\alpha = 140^\circ$

$\alpha = 145^\circ$

$n \backslash m$	0	1	2	3	4	5
1	1.1537	0.8818	1.9307	2.9700	3.9885	4.9958
2	2.3528	1.9510	2.8580	3.8505	4.8383	5.9709
3	3.5834	3.2645	3.9888	4.8528	5.8660	6.8095
4	4.8142	4.5764	5.2004	6.0561	6.8557	7.8520
5	6.0487	5.8819	6.4260	7.2340	8.0238	8.8632
6	7.2853	7.1781	7.6000	8.4181	9.1928	10.0451
7	8.5231	8.4724	8.8951	9.6231	10.7727	11.8199
8	9.7618	9.7174	9.8751	10.6291	11.4107	12.2127
9	11.0010	10.4078	11.1015	11.8362	12.6072	13.4108
10	12.2406	11.6445	12.3310	13.0550	13.8125	14.6002

Table 2.29: Roots λ'_{nm} of the derivative of the associated Legendre Functions with respect to the degree for $\alpha = 145^\circ$

$\alpha = 150^\circ$

$n \backslash m$	0	1	2	3	4	5
1	1.1156	0.9005	1.9567	2.9862	3.9960	4.9989
2	2.2780	1.8857	2.8796	3.8355	4.7915	5.7593
3	3.4480	2.8840	3.8674	4.8099	5.7220	6.6999
4	4.6458	4.1334	4.9423	5.8595	6.8640	7.9174
5	5.8778	5.3043	6.0684	6.9210	7.8551	8.8603
6	7.0321	6.4856	7.2209	8.0302	8.9077	9.8521
7	8.2279	7.6725	8.3879	9.1672	10.0046	10.8986
8	9.4246	8.8630	9.5637	10.3209	11.1292	11.9860
9	10.6220	10.0557	10.7451	11.4854	12.2715	13.1006
10	11.8199	11.2499	11.9303	12.6571	13.4258	14.2333

Table 2.30: Roots λ'_{nm} of the derivative of the associated Legendre Functions with respect to the degree for $\alpha = 150^\circ$

$\alpha = 155^\circ$

$n \backslash m$	0	1	2	3	4	5
1	1.0825	0.9223	1.9768	2.9949	3.9989	4.9998
2	2.2045	1.8783	2.9177	3.9716	4.9924	5.9982
3	3.3435	2.9229	3.8673	4.9220	5.9713	6.9914
4	4.4909	4.0219	4.8763	5.9404	6.9287	7.9728
5	5.6428	5.1466	5.9407	6.8582	7.8783	8.9361
6	6.7973	6.2843	7.0390	7.8959	8.8627	9.8882
7	7.9535	7.4292	8.1569	8.9707	9.8628	10.8543
8	9.1109	8.5784	9.2867	10.0694	10.9245	11.8552
9	10.2690	9.7305	10.4239	11.1834	12.0056	12.8925
10	11.4278	10.8846	11.5663	12.3076	13.1048	13.9578

Table 2.31: Roots λ'_{nm} of the derivative of the associated Legendre Functions with respect to the degree for $\alpha = 155^\circ$

$\alpha = 160^\circ$

$n \backslash m$	0	1	2	3	4	5
1	1.0544	0.9453	1.9898	2.9985	3.9988	5.0000
2	2.1396	1.8897	2.9572	3.9911	4.9985	5.9998
3	3.2405	2.8876	3.9069	4.9705	5.9935	6.9987
4	4.3496	3.9357	4.8659	5.9329	6.9804	7.9955
5	5.4635	5.0135	5.8706	6.8887	7.9546	8.9872
6	6.5804	6.1076	6.9087	7.8607	8.9164	9.9702
7	7.6992	7.2111	7.9728	8.8636	9.8775	10.9415
8	8.8203	8.3203	9.0537	9.8966	10.8554	11.9042
9	9.9436	9.4333	10.1426	10.9343	11.8539	12.8697
10	11.0624	10.5488	11.2447	12.0238	12.8896	13.8519

Table 2.32: Roots λ'_{nm} of the derivative of the associated Legendre Functions with respect to the degree for $\alpha = 160^\circ$

$\alpha = 165^\circ$

$n \backslash m$	0	1	2	3	4	5
1	1.0316	0.9671	1.9966	2.9997	4.0000	5.0000
2	2.0844	1.8859	2.9844	3.9962	4.9998	6.0000
3	3.1493	2.8870	3.9583	4.9935	5.9992	6.9999
4	4.2233	3.8874	4.9352	5.9816	6.9973	7.9996
5	5.3011	4.9177	5.8874	6.9627	7.9954	8.9989
6	6.3822	5.9656	6.8640	7.9336	8.9820	9.9971
7	7.4656	7.0264	7.8712	8.9352	9.9685	10.9824
8	8.5505	8.0951	8.9051	9.9732	10.9565	11.9762
9	9.6365	9.1691	9.9348	10.8596	11.9160	12.9762
10	10.7233	10.2467	10.9861	11.8644	12.8863	13.9568

Table 2.33: Roots λ'_{nm} of the derivative of the associated Legendre Functions with respect to the degree for $\alpha = 165^\circ$

$\alpha = 170^\circ$

$n \backslash m$	0	1	2	3	4	5
1	1.0145	0.9849	1.9993	3.0000	4.0000	5.0000
2	2.0405	1.9572	2.9967	3.9998	5.0000	6.0000
3	3.0748	2.9250	3.9905	4.9993	6.0000	7.0000
4	4.1149	3.8988	4.9790	5.9981	6.9999	8.0000
5	5.1592	4.8859	5.9615	6.9958	7.9996	9.0000
6	6.2065	5.8877	6.9386	7.9908	8.9991	9.9999
7	7.2560	6.9024	7.9133	8.9831	9.9980	10.9998
8	8.3072	7.9271	8.8903	9.9714	10.9961	11.9996
9	9.3596	8.9591	9.8742	10.9554	11.9928	12.9991
10	10.4130	9.9967	10.8677	11.9354	12.9678	13.9983

Table 2.34: Roots λ_{nm} of the derivative of the associated Legendre Functions with respect to the degree for $\alpha = 170^\circ$

$\alpha = 175^\circ$

$n \backslash m$	0	1	2	3	4	5
1	1.0037	0.9962	2.0000	3.0000	4.0000	5.0000
2	2.0109	1.9886	2.9998	4.0000	5.0000	6.0000
3	3.0211	2.9774	3.9994	5.0000	6.0000	7.0000
4	4.0338	3.9634	4.9985	6.0000	7.0000	8.0000
5	5.0487	4.9477	5.9971	6.9999	8.0000	9.0000
6	6.0655	5.9315	6.9949	7.9998	9.0000	10.0000
7	7.0837	6.9163	7.9916	8.9997	10.0000	11.0000
8	8.1032	7.9035	8.9871	9.9994	11.0000	12.0000
9	9.1238	8.8939	9.9812	10.9989	12.0000	13.0000
10	10.1454	9.8880	10.9738	11.9982	12.9999	14.0000

Table 2.35: Roots λ_{nm} of the derivative of the associated Legendre Functions with respect to the degree for $\alpha = 175^\circ$